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Discrepancies in AISC Elastic and Plastic Section Moduli for Unsymmetrical Section

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Abstract

Geometric properties i.e., elastic and plastic section moduli of unsymmetrical sections is determined through a method used by the American Institute of Steel Construction, AISC. This method is based on the assumption that neutral axis always remains parallel to the legs of sections, however, in reality when an unsymmetrical section is used as a flexural member, and load is applied about an axis parallel to one of the legs of the section, biaxial bending is produced in the section which adjusts the placement of neutral axis to a state where the neutral axis does not necessarily continue to be parallel to the legs of section. AISC uses the same procedure for the design of unsymmetrical steel sections as that for symmetrical sections. As design is usually based upon their geometric properties like elastic and plastic section moduli, moment of inertia, etc., so AISC method yields unsafe and uneconomical results. Therefore, unsymmetrical sections should be designed using the author's recommendations based upon their actual analysis. This paper deliberates the incorrect use of values of the AISC in the given circumstances and presents an alternative procedure for the easy and correct design of unsymmetrical sections. This involves the use of equivalent section moduli, Sx, and Zx including the effect of lateral bending.

Keywords: Elastic Section Modulus, Plastic Section Modulus, Unsymmetrical Sections.

1. Introduction

In civil engineering, structures like buildings, bridges, and various sections are used as shear and flexural members. Depending upon the imposed demand we decide sections of various materials to fulfill the design requirement. One of the most important sections is the steel section. The steel section used may be of different shapes like angle section, box section, T section, etc. depending on their manufacturer. Different steel sections are used for different purposes depending upon the demand. The factor to be considered in the design use of these sections is the factor of safety. This means whichever section used its capacity should always be greater than or equal to the applied demand.

A section will be called a symmetrical section if its one portion is exactly the mirror image of other portion when it is cut around the axis (axis of symmetry). The cut section divides the section in two equal parts. A section will be called unsymmetrical, if a given section is not exactly the mirror image of the section that exists on opposite face of axis when cut on the location of axis of symmetry [1].

Symmetrical sections undergo always uniaxial bending whereas unsymmetrical sections undergo bi-axial bending because of this bending stress are produced at various points in the sections.

For most of the times, an eccentric loading is applied to an angle section steel beam in a plane inclined to principal planes. Due to this type of applied load, beam is subjected to primary bending and shear about both principal axes, bending at the supports and torsion.

Even in the very unusual situation when only one of these actions occurs, reduction in the first yield capacities is possible due to local or lateral buckling effects or increased by stiffening resistances which becomes very important at large rotations [2][3].

With reference to principal or geometrical axes, external loading may be applied in the plane of one axis or in most cases two axes. Purlin beam of trusses subjected to biaxial bending is an example of such type of loading. Furthermore, restraining the lateral torsion buckling of angles can lead to achieve more economical solution. Lateral torsional buckling restraining can be done either along the length of the beam or at the section where bending moment is maximum. Biaxial bending on the angle section is caused due to this type of restraining. In case of angle sections, the shear center is not located at the centroid. This results in increased importance of section configuration in calculating critical lateral buckling moment values for the lateral torsion buckling of unequal angles. In other words, it can be said that when long and short legs of angles are under tensile or compressive stresses, it requires different analyses for such type of cases. Theoretical and experimental research has been done and design proposals are given to calculate the strength of angles under bending moments and axial forces [4]– [9].

Geometric properties, i.e., elastic and plastic section moduli about the geometric axes (X and Y axes) are given in LRFD Manual [10] but all these properties are given about geometric axis not about the principal axes. To calculate bending strength about the principal axes, these properties are also required about the principal axes which are not given in LRFD Manual. Thus, calculating these properties about geometric axes is the need of the day [11]. Structural member can be subjected to loading in a plane other than principal plane, for example, an angle section having one leg vertical and loads are applied in vertical plane. Under these conditions, equations used for symmetrical sections are not applicable because the material of the angle is unsymmetrically disposed about the plane of the bending moment [12].

If the plane of loading or plane of bending is not in parallel to the plane that incorporates the principal centroidal axes of cross-section, the bending is known as unsymmetrical bending [13]. Unsymmetrical members about the vertical axes that are generally composed of thin unsymmetrical sections (e.g., L section, Z section, Channel section) go through the phenomenon of twisting underneath the transverse loads. An unsymmetrical section supporting the transverse load might twist because the line of action of the load does not pass via shear center of the member. while rectangular beam would not twist because the loading might bypass via the center of gravity of the section and for such axis symmetrical section the shear center could coincide with the center of gravity of the section. If one desires to apply unsymmetrical sections to hold the transverse load without twisting, it's might possible to accomplish that via locating the load so that it passes through the shear center of the beam. The unsymmetrical sections are usually not used as a flexural member as compared to the symmetrical section because of the possible twisting in the section due to the loading applied transversely on the section.

According to AISC both symmetrical and unsymmetrical steel sections are assumed to be loaded about the principal centroidal axis such that the neutral axis always remains parallel to one of the legs of section [10]. But in reality, an unsymmetrical section used as a flexural member is loaded about an axis other than the principal centroidal axes, the position of the neutral axis shifts such that it may not remain parallel to any leg of the section. This results in biaxial bending. Simultaneous biaxial bending about both principal axes is caused when the loads applied to angle beams usually act out of the principal planes. Biaxial bending is produced in the section if;

- The load-line is inclined to both the principal axes and the section is symmetric about both axes such as circular section, rectangular section, I section etc.
- The load-line is along any vertical centroidal axes and section itself is unsymmetrical such as Z section, angle section or a channel section.

AISC uses Equations (1) and (2) for finding the elastic and plastic section modulus about the x-axis [10],

$$Sx = Ix/c \quad (1)$$

$$Zx = \sum Ay \quad (2)$$

Equations-(1) and (2) are directly applicable only to the design of singly and doubly symmetrical sections e.g., wide flange, Tee, Channels, Double Angles, etc. which are usually loaded in flexure about one of the principal axis and where the neutral axis coincides with the principal axis.

Unsymmetrical sections are usually loaded in flexure about an axis parallel to one of the legs of the section which does not coincide with any of the principal axes. Under such loading conditions, biaxial bending is induced in the section which changes the position of the neutral axis. In this case, analysis can be carried out by applying the detailed expressions of unsymmetrical bending involving moment of inertia about x-axis (Ix), moment of inertia about y-axis (Iy) and product of moment of inertia (Ixy) and the use of elastic section modulus (Sx) and plastic section modulus (Zx) alone cannot provide a correct answer. However, AISC uses equation (1) and (2) for finding the elastic and plastic section moduli of unsymmetrical sections [10], therefore the use of AISC section moduli values in the design of unsymmetrical sections would lead to erroneous results.

The objectives of this paper are to develop a detailed methodology for finding elastic, plastic section moduli, and other geometric properties for unsymmetrical sections without depending upon the formulation of AISC for this section, that considers the same formulation for symmetrical as well as unsymmetrical sections. Also, I have designed unsymmetrical sections based on their actual section moduli.

1.1. AISC Procedure for Calculating Section Moduli:

1.2. Elastic ection Moduli:

According to AISC, to calculate the elastic section modulus about the x-axis (Sx) for L4x3x1/4 and Z6x31/2x3/8, first of all, we have to calculate the moment of inertia about the x-axis (Ix) which is 2.80 in^4 and 25.30 in^4, and then find the distance from the neutral axis to the extreme of the fibre (c) which is 2.764 in and 3 in. So, by putting these values in equation (1) we get elastic section modulus of 1.00 in^3 and 8.40 in^3. While calculating the plastic section modulus about the x-axis (Zx) for L4x3x1/4 and Z6x31/2x3/8, we must first calculate the distance of the neutral axis from the bottom of the fibre which is 0.625 in and 3 in, and then divide the section into smaller areas and find the distance of these smaller areas from the neutral axis and by plugging these values in equation (2) we get plastic section modulus of 1.816 in^3 and 9.66 in^3.

Table 1. Elastic and Plastic Section Modulus of angle and Z sections by AISC Procedure [10].

S.No	Section	Moment of inertia (Ix)	Elastic section modulus (Sx)	Plastic section modulus (Zx)
1	L4x3x1/4	2.80 in^4	1.00 in^3	1.816 in^3
2	Z6x31/2x 3/8	25.30 in^4	8.40 in^3	9.66 in^3

1.3. Authors Recommended Procedure for Calculating Section Moduli:

The procedure to calculate the equivalent elastic section modulus (Sx) for unsymmetrical sections is based on calculating the maximum flexural stresses by mechanics-based expressions for biaxial bending and then back calculating elastic section modulus (Sx) The formulation ignores the effect of lateral bracing in the modification of lateral bending behavior. Further, it is assumed that the load is applied through the shear center and torsion is not produced.

The stress at any point of a section subjected to a moment about x-axis can be calculated as [14]:

$$\sigma=\left(\frac{-M_x I_{xy}}{I_y I_x - I_{xy}^2}\right)x + \left(\frac{M_x I_y}{I_y I_x - I_{xy}^2}\right)y \qquad (3)$$

Where x and y are the distances of each point on the given unsymmetrical section from the neutral axis. Their values may be positive as well as negative; σ is the stress at any point of an unsymmetrical section; Mx is the bending moment about the x-axis; Ix is the moment of inertia about x-axis; Iy is the moment of inertia about y-axis; Ixy is the product of inertia.

Equation (3) can also be re-written in the form

$$\frac{\sigma}{M_x} = \left(\frac{-I_{xy}}{I_y I_x - I_{xy}^2}\right)x + \left(\frac{I_y}{I_y I_x - I_{xy}^2}\right)y = \frac{I_y y - I_{xy} x}{(I_y I_x - I_{xy}^2)} \qquad (4)$$

The equation of neutral axis can be obtained by substituting zero for σ in Equation (3), which after rearranging becomes,

$$y = (Ixy/Iy)x \qquad (5)$$

1.1.1. Elastic Section Modulus of Angle Sections:

The elastic neutral axis of a typical angle section is shown in Fig-1. Various corners of the angle section are labeled, starting at the top left corner and moving in the clockwise direction.

To locate the point at which first yielding starts, the value of σ/Mx is calculated at various corner points of the angle section, e.g., points 1,2,3,4 & 5. The absolute maximum value is noted which corresponds to the first yielding. Reciprocal of the value of equation (4) gives the elastic section modulus for moment about the x-axis (Sx).

$$S_x = \left|\frac{M_x}{\sigma}\right| \min \qquad (6)$$

The maximum value of σ/Mx usually corresponds to point 2 i-e, first yielding starts at point 2. A spreadsheet has been developed to calculate the elastic section modulus of the given angle section.

Based on the author's recommendation, for determining the elastic section modulus of Angle Section L4x3x1/4 (Fig-2) about an axis parallel to the shorter leg, we must first determine the geometric properties in which distances of extremes of fibre from the neutral axis are 0.736 in from y-axis and 1.236 in from x- axis, the product of moment of inertia is -1.146 in4, moment of inertia about x and y axes are 2.769 in4 and 1.355 in4.

By putting these values in Equation-(4), equation of fiber stress is;

$$\sigma / Mx = (0.470)\ x + (0.555)\ y \quad (7)$$

Table 2: Elastic Section Modulus of angle section by author method:

Point	x	y	σ / Mx
1	-0.736	+2.764	+1.189
2	-0.486	+2.764	+1.307
3	+2.264	-0.986	+0.516
4	+2.264	-1.236	+0.377
5	-0.736	-1.236	-1.032

From the above table, it is clear that yielding starts at point-2 i.e., the value of σ/Mx is maximum at this point, and the value of σ/M_x is 1.307 in-3. As the equivalent elastic section modulus is the reciprocal of σ/M_x, so the value of elastic section modulus is 0.765 in^3.

1.1.2. Elastic section modulus of Z section:

The elastic neutral axis of a typical Z section is shown in Fig-3. Various corners of the Z section are labeled, starting at the top left corner and moving in the clockwise direction.

According to the author's recommendation, for finding the elastic section modulus of Z Section Z6x31/2x3/8 (Fig-4), we have to find the geometric properties in which distances of extreme of fibre from the neutral axis are 3.3125 in from y-axis and 3 in from x-axis, the product of moment of inertia is -11.536 in4, moment of inertia about x and y-axis are 25.3 in4 and 9.1 in4.

By putting these values in Equation-(4), equation of fiber stress is,

$$\sigma / M_x = (0.119)\ x + (0.0937)\ y \quad (8)$$

Table 3: Elastic Section Modulus of Zee section by author method:

Points	X	Y	σ/M
1	-3.3125	3	-0.113
2	0.1875	3	0.303
3	0.1875	-2.625	-0.224
4	3.3125	-2.625	0.148
5	3.3125	-3	0.113
6	-0.1875	-3	-0.303
7	-0.1875	2.625	0.224
8	-3.3125	2.625	-0.148

From the above table, it is clear that yielding starts at point-2 and 6 i.e., the value of σ/Mx is maximum at points 2 and 6. Therefore, the value of σ/Mx is 0.303 in-3, and as the equivalent elastic section modulus is the reciprocal of σ/Mx, the value of elastic section modulus is 3.3 in^3.

1.3 Plastic Section Modulus:

In case of plastic section modulus, the neutral axis coincides with the equal area axis of the section. As already discussed singly or doubly symmetrical sections are usually loaded about one of the principal axes, the neutral axis remains parallel to the principal axis about which the moment is applied. An unsymmetrical section like angle section is usually loaded in flexure about an axis other than the principal axis which changes the position of the neutral axis to a state where it does not necessarily remain parallel to the legs of the section.

The plastic section modulus of any section may be defined as the first moment of area of section about plastic neutral axis.

1.2.1. Plastic section modulus of Angle section:

To calculate the plastic section modulus, the angle section is divided into a number of sub-areas as shown in Fig-5.

The areas and their corresponding centroidal distances from the x and y axes are given below.

Table 4: Derived formulas of plastic section modulus of angle section:

S. No	Areas (Ai)	Centroidal distance (X_i)	Centroidal distance (Y_i)
1	$A_1=\left(H-\frac{c}{m}\right)t$	$x_1=\frac{H}{2}+\frac{c}{2m}$	$y_1=\frac{t}{2}$
2	$A_2=\frac{t^2}{2m}$	$x_2=\frac{c}{2}-\frac{t}{3m}$	$y_2=\frac{2t}{3}$
3	$A_3=-\frac{t^2}{2m}$	$x_3=\frac{c}{m}-\frac{2t}{3m}$	$y_3=\frac{t}{3}$
4	$A_4=-\left(\frac{c-t}{m}\right)t$	$x_4=\frac{c-mt}{2}$	$y_4=\frac{t}{2}$
5	$A_5=-(c-t-mt)t$	$x_5=\frac{t}{2}$	$y_5=\frac{c-t-mt}{2}$
6	$A_6=-\frac{mt^2}{2}$	$x_6=\frac{t}{3}$	$y_6=c-\frac{2mt}{3}$
7	$A_7=\frac{mt^2}{2}$	$x_7=\frac{2t}{3}$	$y_7=c-\frac{mt}{3}$
8	$A_8=(V-mt)t$	$x_8=\frac{t}{2}$	$y_8=\frac{V+c}{2}$

Note: The areas below the neutral axis are written negative in the above equations, e.g. A3, A4, A5, and A6. Now the sum of areas on both sides of the neutral axis are equal and equal to half of the total area.

Therefore,

$$A3+A4+A5+A6=A/2 \quad (9)$$

Putting the corresponding expressions of sub-areas, we get:

$$c=m/(1+m)(A/2t+t+t/2m+mt/2) \quad (10)$$

Now for the plastic analysis, three equilibrium equations must be satisfied, i.e.

$$\sum Fx=0 \quad \sum My=0 \quad \sum Mx=Mx$$

The 1st equilibrium is satisfied if equation (7) is satisfied. To satisfy the 2nd equilibrium equation the first moment of area about y-axis must equal to zero, i.e.

$$\sum Ax=0 \quad (11)$$

The 3rd equilibrium is satisfied when the moment of area about the x-axis is equal to the equivalent plastic section modulus, i.e.

$$Zx = \sum Ay \qquad (12)$$

Now to calculate the equivalent plastic section modulus of angle section for moment about x-axis, a spreadsheet may be developed. The procedure is summarized in steps as follows:

1. For an assumed value of θ, the value of m is calculated.
2. Using equation (8) value of c is calculated.
3. First moment of area about the y-axis is calculated.
4. The value of θ is changed till equation (9) is satisfied.

For the value of θ satisfying equation (9), the plastic section modulus is calculated using Equation (10).

Based on the author's recommendation, for determining the plastic section moduli of Angle Section L4x3x1/4 (Fig-6) about an axis parallel to the shorter leg, first of all, we have to find the value of m which is equal to tanθ, and from trial and error or using a spreadsheet the inclination of neutral axis can easily be found, and which is 33.410, so from this the value of m will be 0.660. For finding the value of c we will put the value of area as 1.6875 in2 and m equal to 0.660 in Equation (8), so we get the value of c as 1.549in.

Using the expression of sub-areas and their centroidal distances we can develop the following Table-5:

Table 5: Plastic Section Modulus of Angle section:

S.NO	A	Y	AY	X	AX
1	0.163	0.125	0.020	2.674	0.436
2	0.047	0.167	0.008	2.222	0.105
3	-0.047	0.083	-0.004	2.095	-0.099
4	-0.492	0.125	-0.062	0.985	-0.485
5	-0.284	0.817	-0.232	0.125	-0.035
6	-0.021	1.439	-0.030	0.083	-0.002
7	0.021	1.494	0.031	0.167	0.003
8	0.613	2.774	1.700	0.125	0.077
	∑A=0		Zx=∑Ay=1.432		Zy =∑Ax = 0

Therefore, the equivalent plastic section modulus of above angle section is 1.432 in^3.

1.2.2. Plastic section modulus of Z section:

To calculate the plastic section modulus, the Z section is divided into a number of sub-areas as shown in Fig-7.

The areas and their corresponding centroidal distances from the x and y axes are given below:

Table 6: Derived formulas of plastic section modulus of Z section:

S. No	Areas	x centroidal distance	y centroidal distance
1	$A_1 = \frac{-(2mH - mt - V) * t}{2m}$	$X_1 = \frac{-(2mH - mt + V)}{4m}$	$Y_1 = \frac{(V - t)}{2}$
2	$A_2 = \frac{-t^2}{2m}$	$X_2 = \frac{2t - 3V}{6m}$	$Y_2 = \frac{3V - 4t}{6}$
3	$A_3 = \frac{t^2}{2m}$	$X_3 = \frac{4t - 3V}{6m}$	$Y_3 = \frac{3V - 2t}{6}$
4	$A_4 = \frac{(V - 2t + mt) * t}{2m}$	$X_4 = \frac{(2t + mt - V)}{4m}$	$Y_4 = \frac{(V - t)}{2}$
5	$A_5 = \frac{(V - 2t - mt) * t}{2}$	$X_5 = 0$	$Y_5 = \frac{V - 2t + mt}{4}$
6	$A_6 = \frac{mt^2}{2}$	$X_6 = \frac{t}{6}$	$Y_6 = \frac{mt}{6}$
7	$A_7 = \frac{-mt^2}{2}$	$X_7 = \frac{-t}{6}$	$Y_7 = \frac{-mt}{6}$
8	$A_8 = \frac{-(V - 2t - mt) * t}{2}$	$X_8 = 0$	$Y_8 = \frac{-(V - 2t + mt)}{4}$
9	$A_9 = \frac{-(V - 2t + mt) * t}{2m}$	$X_9 = \frac{-(2t + mt - V)}{4m}$	$Y_9 = \frac{-(V - t)}{2}$
10	$A_{10} = \frac{-t^2}{2m}$	$X_{10} = \frac{-(4t - 3V)}{6m}$	$Y_{10} = \frac{-(3V - 2t)}{6}$
11	$A_{11} = \frac{t^2}{2m}$	$X_{11} = \frac{(3V - 2t)}{6m}$	$Y_{11} = \frac{(4t - 3V)}{6}$
12	$A_{12} = \frac{(2mH - mt - V) * t}{2m}$	$X_{12} = \frac{(2mH - mt + V)}{4m}$	$Y_{12} = \frac{-(V - t)}{2}$

$m = \tan\theta$

The areas below the neutral axis are written negative in the above equations, e.g. A1, A2, A7, A8, A9, and A10 are written negative.

According to the author's recommendation, for determining the plastic section modulus of Z section Z6x31/2x3/8 (Fig-8) we must first determine the value of m, so for m we have to find the θ and from trial and error or using a spreadsheet the inclination of neutral axis can easily be found, so the value of θ is 49.260, and from this, the value of m is 1.161.

Table 7: Plastic Section Modulus of Z section:

S. No	A	x	Ax	y	Ay
1	-0.273	-2.948	0.806	2.813	-0.769
2	-0.061	-2.476	0.150	2.75	-0.167
3	0.061	-2.368	-0.143	2.875	0.174
4	0.918	-1.037	-0.952	2.813	2.582
5	0.903	0	0.000	1.421	1.283
6	0.082	0.063	0.005	0.073	0.006
7	-0.082	-0.063	0.005	-0.073	0.006
8	-0.903	0	0.000	-1.421	1.283
9	-0.918	1.037	-0.952	-2.813	2.582
10	-0.061	2.368	-0.143	-2.875	0.174
11	0.061	2.476	0.150	-2.75	-0.167
12	0.364	2.948	1.074	-2.813	-1.025
Σ A	**0.091**	**Σ Ax**	**0.000**	**Σ Ay**	**5.963**

Therefore, the equivalent plastic section modulus of above Z section is 5.963 in^3.

2. Results and Discussions:

The elastic and plastic section modulii about the x-axis of few angle and Z sections as calculated by AISC procedure and the equivalent section modulii as recommended by authors are listed in the following tables:

Table 8: Comparison of elastic section modulii of angle section as recommended by the authors and AISC:

S. No	Angle section	Elastic section modulus about x-axis (in^3)		Ratio of AISC procedure and author method
		As recommended by the author [14]	As calculated by AISC procedure [10]	
1	L2x2x1/8	0.102	0.131	**1.28**
2	L2x2x1/4	0.189	0.247	**1.31**

3	L3x2x1/4	0.406	0.542	**1.33**
4	L3x3x1/4	0.445	0.577	**1.30**
5	L4x3x1/4	0.765	1.00	**1.31**
6	L4x4x1/4	0.814	1.05	**1.29**
7	L5x3x1/4	1.17	1.53	**1.31**
8	L5x5x1/2	2.43	3.16	**1.30**
9	L6x6x1	6.50	8.57	**1.32**
10	L8x8x1	12.06	15.80	**1.31**

Table 9: Comparison of plastic section modulii of angle section as recommended by the authors and AISC:

S. No	**Angle section**	**Plastic section modulus about x-axis (in^3)**		**Ratio of AISC procedure and author method**
		As recommended by the author [14]	**As calculated by AISC procedure** [10]	
1	L2x2x1/8	0.195	0.235	**1.205**
2	L2x2x1/4	0.370	0.445	**1.203**
3	L3x2x1/4	0.764	0.973	**1.274**
4	L3x3x1/4	0.862	1.04	**1.206**
5	L4x3x1/4	1.43	1.82	**1.273**
6	L4x4x1/4	1.56	1.88	**1.205**
7	L5x3x1/4	2.13	2.72	**1.277**
8	L5x5x1/2	4.72	5.68	**1.203**
9	L6x6x1	12.92	15.46	**1.197**
10	L8x8x1	23.69	28.47	**1.202**

Table 10: Comparison of elastic section modulii of Z section as recommended by the authors and AISC:

S. No	Z section	Elastic section modulus about x-axis (in^3)		Ratio of AISC procedure and author method
		As recommended by the author [14]	As calculated by AISC procedure [10]	
1	Z61/8x35/8x1/2	4.369	11.24	**2.573**
2	Z 6x7/2x3/8	3.31	8.44	**2.550**
3	Z51/8x33/8x7/16	2.8	7.44	**2.657**
4	Z51/16x35/16x3/8	2.42	6.39	**2.640**
5	Z5x31/4x1/2	2.88	7.68	**2.667**
6	Z5x31/4x5/16	2.03	5.34	**2.631**
7	Z41/8x33/16x3/8	1.68	4.67	**2.780**
8	Z41/16x31/8x1/2	1.98	5.5	**2.778**
9	Z41/16x31/8x5/16	1.42	3.91	**2.754**
10	Z4x31/16x1/4	1.15	3.14	**2.730**

Table 11: Comparison of plastic section modulii of Z section as recommended by the authors and AISC:

S. No	Z section	Plastic Section Modulus about x-axis (in^3)		Ratio of AISC procedure and author method
		As recommended by the author [14]	As calculated by AISC Procedure [10]	
1	Z61/8x35/8x1/2	7.987	13.479	**1.688**
2	Z 6x7/2x3/8	5.964	9.967	**1.671**
3	Z51/8x33/8x7/16	5.173	8.897	**1.720**
4	Z51/16x35/16x3/8	4.422	7.566	**1.711**
5	Z5x31/4x1/2	5.423	9.313	**1.717**
6	Z5x31/4x5/16	3.682	6.256	**1.699**
7	Z41/8x33/16x3/8	3.139	5.55	**1.768**
8	Z41/16x31/8x1/2	3.798	6.739	**1.774**
9	Z41/16x31/8x5/16	2.611	4.585	**1.756**
10	Z4x31/16x1/4	2.09	3.637	**1.740**

3. Conclusions:

The main conclusions can be drawn as follows;

1. The AISC elastic and plastic section moduli percentage ratio of Z sections shows that AISC/Author differ by about 2.78 and 1.774 respectively. Likewise, the percentage ratio of AISC/Author of elastic and plastic section moduli of angle sections differ by about 1.33 and 1.277 respectively. The use of AISC moduli will not only overestimate but also render unsafe results, making the design remarkably conservative.
2. Principal axes are the axes about which the product moment of inertia is zero. Hence, it can be concluded saying that the simple bending theory is applicable for bending moment about principal axes only.
3. AISC procedure is only applicable if uniaxial bending is produced in the section, such as symmetrical sections. For unsymmetrical sections where biaxial bending is produced, the author's recommended method should be adopted.
4. The author recommended equivalent section moduli can quickly and conveniently analyze and design unsymmetrical sections provided the lateral bending pattern is not altered by lateral bracing and the load in bending is applied through the shear center.

4. Future Recommendations:

1. Experimental Study of an angle section loaded in flexural about an axis parallel to one of its legs is recommended to verify the author's recommended section moduli.
2. A study regarding the elastic and plastic section moduli of AISC angle section considering the corners rounding effect will be useful to replace the current values.
3. It is also recommended to study the effect of lateral resistant due to bracing and other adjoining members.
4. To make it more economical and safer it is recommended that unsymmetrical sections should be designed using their actual values of section moduli by author recommendation.

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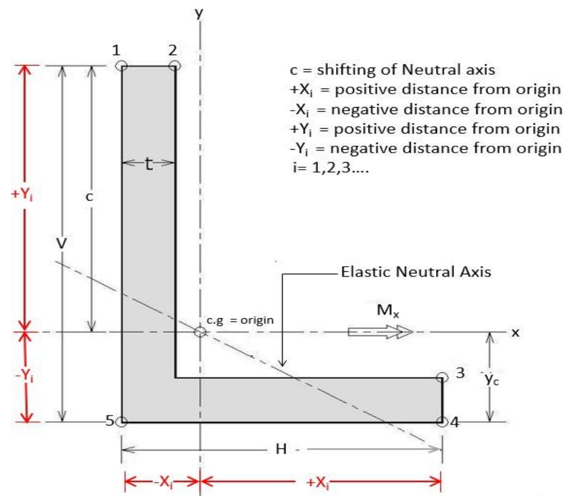

Fig-1: Orientation of Elastic Neutral Axis of Typical Angle Section as recommended by the Authors

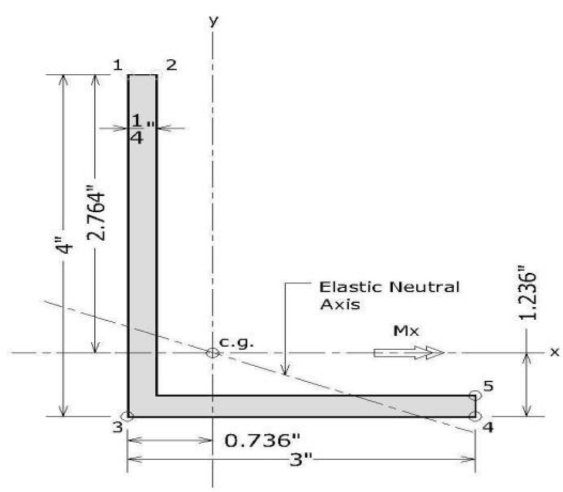

Fig-2: Orientation of Elastic Neutral Axis of L4x3x1/4 as recommended by the Authors

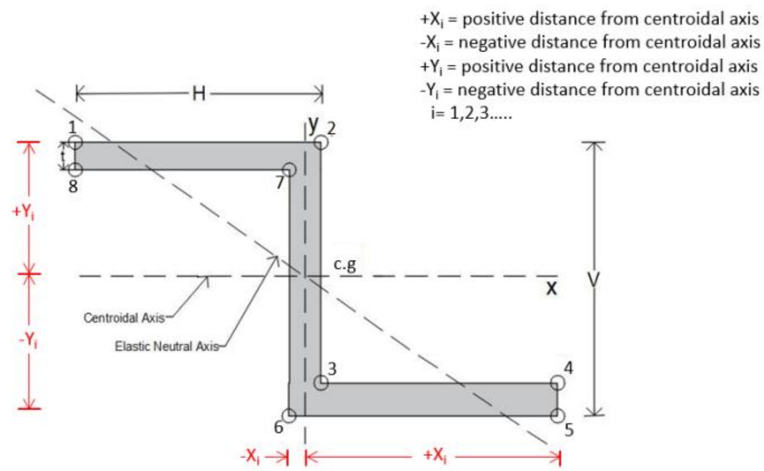

Fig-3: Orientation of Elastic Neutral Axis of Typical Z Section as recommended by the Authors

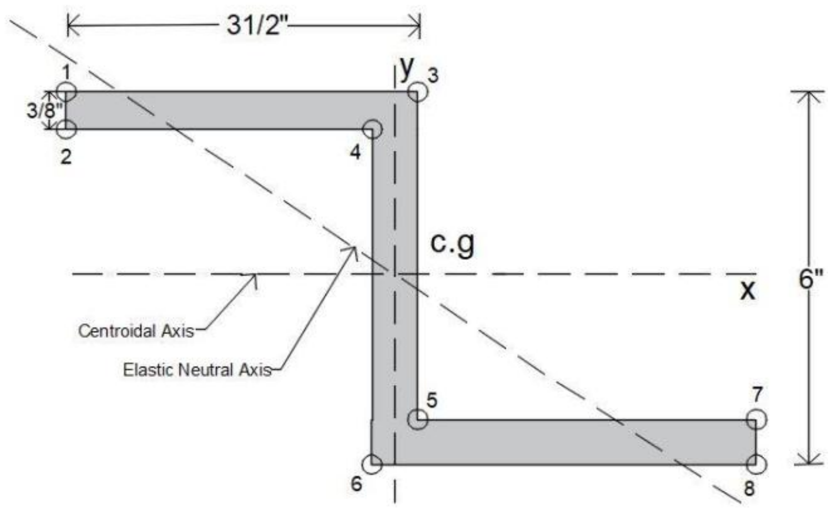

Fig-4: Orientation of Elastic Neutral Axis of Z6x31/2x3/8 as recommended by the Authors

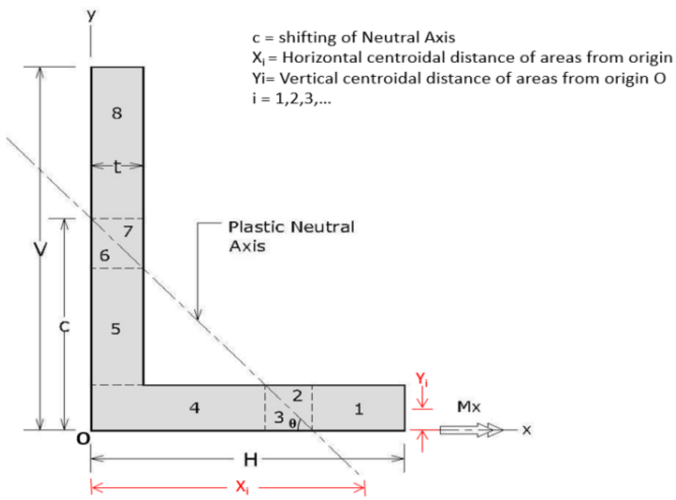

Fig-5: Orientation of Plastic Neutral Axis of Typical Angle Section as recommended by Authors

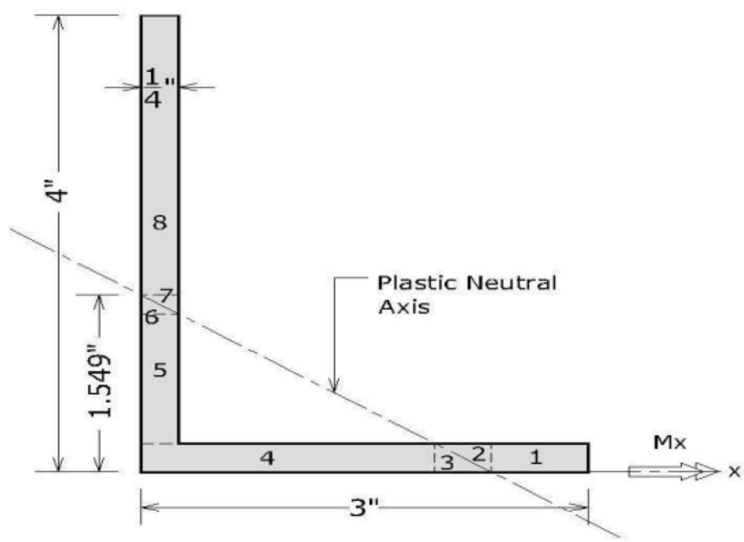

Fig-6: Orientation of Plastic Neutral Axis of L4x3x1/4 as recommended by the Authors

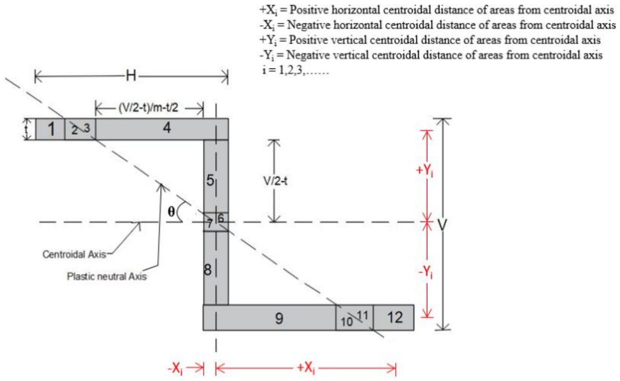

Fig-7: Orientation of Plastic Neutral Axis of Typical Z Section as recommended by the Authors

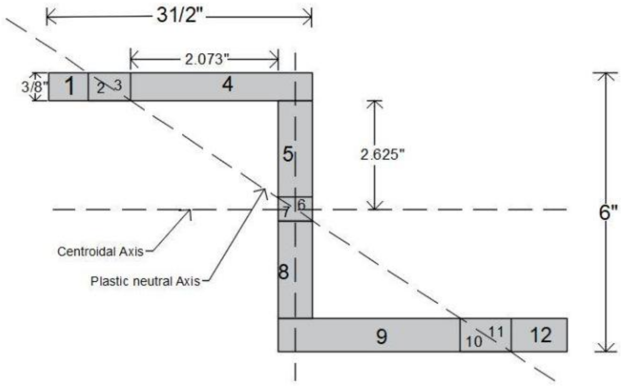

Fig-8: Orientation of Plastic Neutral Axis of Z6x31/2x3/8 as recommended by the Authors